1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 01 (Physics Part-1, Class XI)
Module Name/Title	Unit 2, Module 6, **Vectors** Chapter 4: Motion in a plane
Module Id	Keph_10401_eContent
Pre-requisites	Kinematics of motion in one dimension, basic trigonometry, basic geometry
Objectives	After going through this module, the learners will be able to: • Understand vector algebra and its relation to physical quantities • Apply methods of vector addition (including graphical methods) • Resolve vectors into component and learn another method of vector addition • Distinguish between scalar product and vector product
Keywords	Vector, addition of vectors, resolution of vector, product of vector, Scalar product, vector product

2. Development Team

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1. UNIT SYLLABUS

Chapter 3: Motion in a straight line

Frame of reference, motion, position –time graph Speed and velocity
Elementary concepts of differentiation and integration for describing motion, uniform and non-uniform motion, average speed and instantaneous velocity, uniformly accelerated motion, velocity-time and position time graphs relations for uniformly accelerated motion - equations of motion (graphical method).

Chapter 4: Motion in a plane

Scalar and vector quantities, position and displacement vectors, general vectors and their notations, multiplication of vectors by a real number , addition and subtraction of vectors, relative velocity, unit vector, resolution of a vector in a plane, rectangular components, scalar and vector product of vectors
Motion in a plane, cases of uniform velocity and uniform acceleration projectile motion uniform circular motion.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 10 Modules

The above unit is divided into 10 modules for better understanding.

Module 1	• Introduction to moving objects • Frame of reference, • limitations of our study • treating bodies as point objects
Module 2	• Motion as change of position with time • Distance travelled unit of measurement • Displacement negative, zero and positive • Difference between distance travelled and displacement • Describing motion by position time and displacement time graphs
Module 3	• Rate of change of position • Speed • Velocity • Zero , negative and positive velocity • Unit of velocity • Uniform and non-uniform motion • Average speed • Instantaneous velocity • Velocity time graphs • Relating position time and velocity time graphs
Module 4	• Accelerated motion • Rate of change of speed, velocity • Derivation of Equations of motion
Module 5	• Application of equations of motion • Graphical representation of motion • Numerical
Module 6	• Vectors

	• Vectors and physical quantities • Vector algebra • Relative velocity • Problems
Module 7	• Motion in a plane • Using vectors to understand motion in 2 dimensions' projectiles • Projectiles as special case of 2 D motion • Constant acceleration due to gravity in the vertical direction zero acceleration in the horizontal direction • Derivation of equations relating horizontal range vertical range velocity of projection angle of projection
Module 8	• Circular motion • Uniform circular motion • Constant speed yet accelerating • Derivation of $a = \frac{v^2}{r} or\ \omega^2 r$ • direction of acceleration • If the speed is not constant? • Net acceleration
Module 9	• Numerical problems on motion in two dimensions • Projectile problems
Module 10	• Differentiation and integration • Using logarithm tables

MODULE 6

3. WORDS YOU MUST KNOW

Let us remember the words we have been using in our study of this physics course

- **Frame of reference**: Any reference frame having coordinates(x, y, z), which indicate the change in position of object with time.
- **Motion in two dimension**: When the position of an object can be shown by changes any two coordinate out of the three (x, y, z), also called motion in a plane.
- **Motion in three dimension**: When the position of an object can be shown by changes in all three coordinate out of the three (x, y, z).
- **Scalar**: Physical quantity that have only magnitude.
- **Vector**: Physical quantity that has both magnitude and direction.
- **Distance covered**: The distance an object has moved from its starting position. The SI unit of distance is 'm'. Distance can be zero or positive.
- **Displacement**: The distance an object has moved from its starting position moves in a particular direction .The SI unit of displacement is 'm'. Displacement can be zero, positive or negative.
- **Path length**: Actual distance covered is called the path length.
- **Speed**: Rate of change of position is called speed and its SI unit is m/s.
- **Velocity:** Rate of change of position in a particular direction is called velocity. It can be zero, negative and positive. Its SI unit is m/s.
- **Acceleration:** rate of change of velocity
- **Equations of motion:** equations relating initial final velocity time and displacement for objects moving with constant acceleration. These can be used to describe motion

4. INTRODUCTION

In physics, we can classify quantities as scalars or vectors. The difference between them is that, a direction is associated with a vector but not with a scalar. A scalar quantity is a quantity with magnitude only. It is specified completely by a single number, along with the proper unit. For example: the distance between two points, mass of an object, the temperature of a body and the time at which a certain event happened.

In our study of physics, we need mathematics.

Most often, Physics uses mathematical tools to analyze, understand, explain and express the relations between various physical quantities.

We developed the concepts of **position, displacement, velocity and acceleration** that are needed to describe the motion of an object along a straight line. We found that the directional aspect of these quantities can be taken care of by positive and negative signs, as in one dimension only two directions are possible. But in order to describe **motion of an object in two dimensions (a plane) or three dimensions (space),** we need to use vectors to describe the above-mentioned physical quantities.

Therefore, it is first necessary to learn the language of vectors.

What is a vector?

How to add, subtract and multiply vectors?

What is the result of multiplying a vector by a real number?

What is the result when we multiply two vectors?

5. VECTORS AND PHYSICAL QUANTITIES

We have learnt about physical quantities. We can classify them in two basic categories.

(*i*) **Scalars: characterized by only a magnitude *e.g.* time, mass speed distance as we learnt in our earlier lesson.**

(*ii*) **Vectors**: **characterized by both a magnitude and a direction *e.g* displacement, Velocity, force**

The rules for combining scalars are the rules of ordinary algebra. Scalars can be added, subtracted, multiplied and divided using algebraic computational methods and operations.

But rules to go the same i.e. add, subtract, and multiply are different for vectors because of the direction associated with it.

However, all physical quantities with magnitude and direction are not necessarily vectors, like current

In this module we will learn basic vector algebra

6. VECTOR ALGEBRA

What is a vector in mathematics?

Like a map is a representation of an area on ground, we can say a vector can represent a physical quantity that has both magnitude and direction.

Geometrically, a vector is represented by a line and an arrow.

How can a vector represent a physical quantity?

The length of a vector represents the magnitude of the vector and the arrow head represents the direction.

Since a vector has two characteristics a magnitude and a direction.

So, a line indicative of its magnitude and an arrow its direction is its geometrical representation. The arrow has a head or a tip and the other end is called the tail of the vector

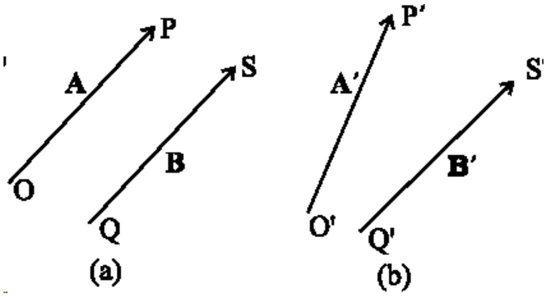
A physical quantity which is vector with a arrow head shown as $\vec{\mathbf{A}}$ having magnitude of vector $\vec{\mathbf{A}}$ is$|A|$.

Before we learn the vector algebra, it is necessary to describe some commonly applied definitions, because we may need to compare and relate two or more vector physical quantities.

EQUAL VECTORS

Two vectors are said to be equal, if both have the same magnitude and the same direction. For example, a velocity of 5 km/h east is not the same as 5 km/h north. Although, the magnitudes are the same, the two velocities have different directions; therefore they are two different vectors.

Consider two equal vectors A and B as shown in the figure (a) & (b) above. We can easily check their equality. Shift B parallel to itself until its tail Q coincides with that of A, i.e. Q coincides with O. Then, since their tips S and P also coincide, the two vectors are said to be equal. In general, equality is indicated as A = B. Note that in vectors A′ and B′ have the same magnitude but they are not equal because they have different directions. Even if we shift B′ parallel to it so that its tail Q′ coincides with the tail O′ of A′, the tip S′ of B′ does not coincide with the tip P′ of A′.



NEGATIVE OF A VECTOR

When two vectors have the same magnitude but opposite directions, they are called the negative of each other. A force of 10 N east and 10 N west are negative of each other.

MODULUS OF VECTOR

The modulus of a vector means the length or magnitude of a vector. It is a scalar quantity

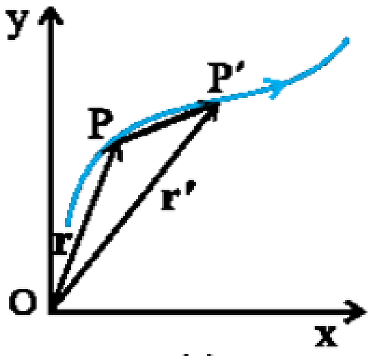
$$|\boldsymbol{A}| = A$$

UNIT VECTOR

A unit vector is a vector of unit magnitude drawn in the direction of a given vector. It is a vector in the direction of a given vector is found by dividing the given vector by its modulus

Unit vector of $\vec{A} = \hat{A} = \frac{\vec{A}}{|A|}$

POSITION VECTOR AND DISPLACEMENT VECTOR



The most basic vector is the position vector. As the name suggests, **is line joining the origin to the position of the object.**

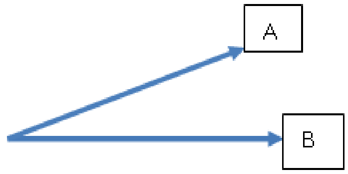
The position vector represents the position of a body with respect to a chosen origin at an instant of time.

To describe the position of an object moving in a plane, we need to choose a convenient point, say O as origin.

Let P and P′ be the positions of the object at time t and t′ respectively, as shown above.
We join O and P by a straight line. Then, OP is the position vector of the object at time t.
An arrow is marked at the head of this line. It is represented by a symbol r, i.e. OP = r.
Point P′ is represented by another position vector, OP′ denoted by r′.
The length of the vector represents the magnitude of the vector and its direction is the direction in which P lies as seen from O. If the object moves from P to P′, the vector PP′ (with tail at P and tip at P′) is called the displacement vector corresponding to motion from point P (at time t) to point P′ (at time t′).

CO INITIAL VECTORS

Two vectors which have the same initial point are called co initial vectors



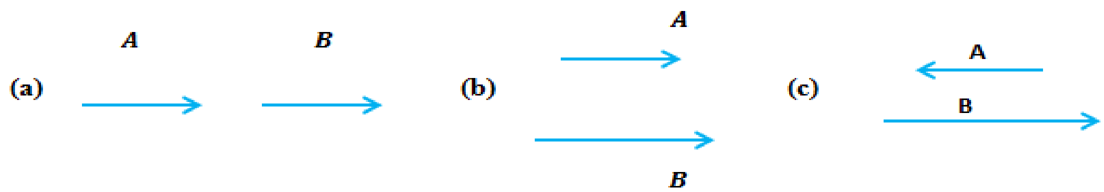
ZERO OR NULL VECTOR

A zero or a null vector has no defined direction and no magnitude. For example, a vector representing a velocity at the highest point when a body is thrown upwards under gravity.

We need the zero vectors or null vectors, if the resultant of addition of vectors is zero.

COLLINEAR VECTORS

The vectors which either act along the same line or along parallel lines are called collinear vectors.



COPLANAR VECTORS

Vectors which are in the same plane

RESULTANT VECTOR:

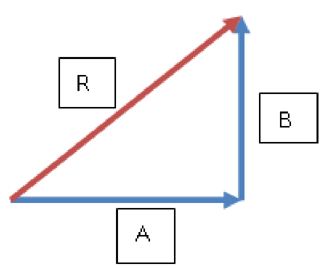
A single vector which can represent all other vectors together or the combined effect of two or more vectors is given by its resultant.

6. VECTOR ALGEBRA

It is the branch of mathematics with rules of computing vector quantities. Since vectors have both magnitude and direction there are special ways of addition, subtraction and multiplication.

VECTOR ADDITION AND SUBTRACTION

A man undergoes a displacement A along east and then a displacement B along north to reach point Q from P. He could have attained the same result had he moved a displacement R.



Thus, R is called the resultant of A and B and we write **R= A + B.**

The sides of the triangle are drawn proportional to the magnitude of displacement A and B

Note that this is not the usual algebraic sum.

There are many ways to add vectors.

- **Each one of them is useful depending upon the number of vectors involved**
- **Each method gives the same result**
- **The graphical method involves actual drawing of geometrical figures according to rules**
- **Analytical method proposes a formula which can be used to find the resultant vector**
- **Resolution of vector means finding components of a vector so that its effect can be described in a particular direction**

The simplest method of adding vectors is the graphical method.

Graphical methods are

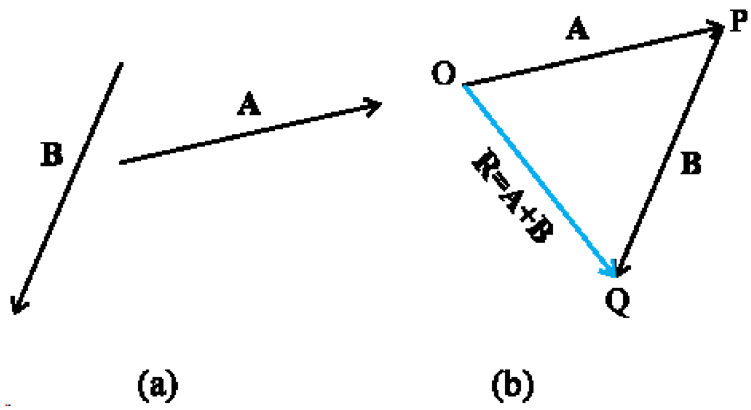
(i) **The triangle Law of Vector Addition**
(ii) **The parallelogram method.**
(iii) **The polygon law of vector addition**

TRIANGLE LAW OF VECTOR ADDITION

If two vectors are represented both in magnitude and direction by two sides of a triangle taken in the same order, the resultant of the two vectors is represented both in magnitude and direction by the third side of the triangle taken in opposite order.

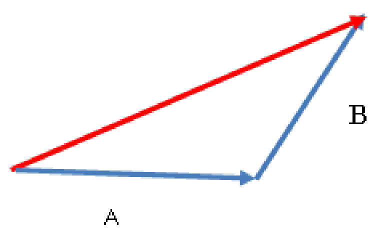
Remember the head and tail of the vector are important. “In order” means the tail of the second vector to coincide with the head of the first vector. “Opposite order’ means the resultant vector will have the direction in such a way that the vector order is reversed.

Let us consider two vectors A and B that lie in a plane as shown below in (a)



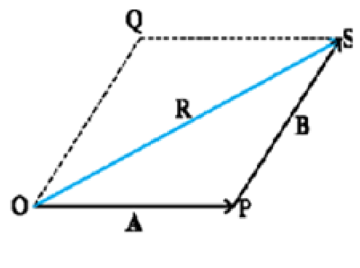
The lengths of the line segments representing these vectors are proportional to the magnitude of the vectors. To find the sum **A + B**, we place vector B so that its tail is at the head of the vector A, as shown above in (b)

Simply stated, we can add two vectors A and B by putting the tail of B at the head of the arrow A, then the resultant will be the vector drawn from the tail of A to the head of arrow B.

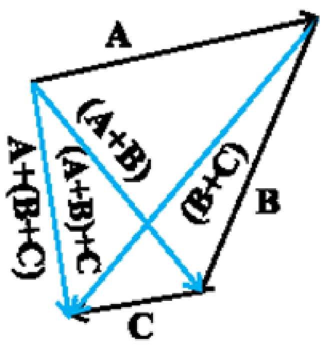


This graphical method is also called the head-to-tail method. The two vectors and their resultant form three sides of a triangle, so this method is also known as triangle method of vector addition.

If we find the resultant of (B + A), as shown below in (a) and (b).



(a)



(b)

Vector addition is commutative. *i.e.*, the order of addition does not change their resultant.

The same vector R is obtained. Thus, vector addition is commutative:

∴ **A + B = B + A**

Thus A + B = B + A (as shown above in (a)).

Vector addition is associative.

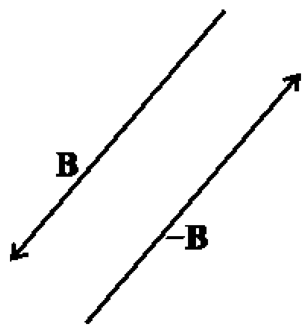
The addition of vectors also obeys the associative law as illustrated above in (b).

The result of adding vectors A and B first and then adding vector C is the same as the result of adding B and C first and then adding vector A :

(A + B) + C = A + (B +C)

What is the result of adding two equal and opposite vectors?

Consider two vectors **B** and **–B** shown below:



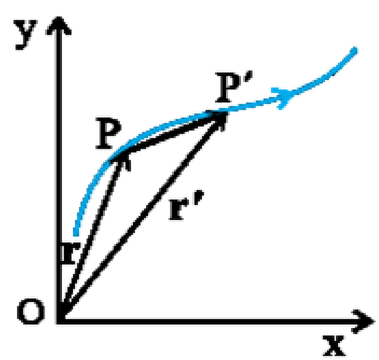
Their sum is **B + (–B**). Since the magnitudes of the two vectors are the same, but the directions are opposite, the resultant vector has zero magnitude and is represented by 0 called a null vector or a zero vector.

B – B = 0, |0| = 0

Since the magnitude of a null vector is zero, its direction cannot be specified.

What is the physical meaning of a zero vector?

Consider the position and displacement vectors in a plane as shown in Fig. here. Now suppose that an object which is at P, at time t, moves to P′ and then comes back to P. Then, what is its displacement? Since the initial and final positions coincide, the displacement is a "null vector"



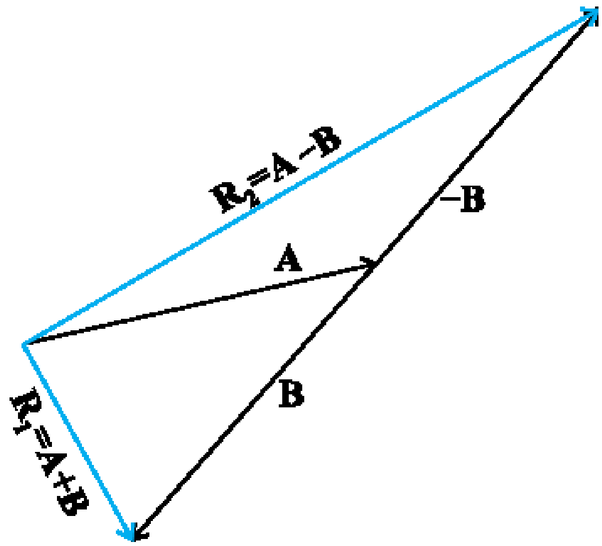
Subtraction of vectors

Subtraction of vectors can be defined in terms of addition of vectors.

We define the difference of two vectors A and B as the sum of two vectors A and –B:

A – B = A + (–B)

It is shown below.



The vector **–B** is added to vector **A** to get

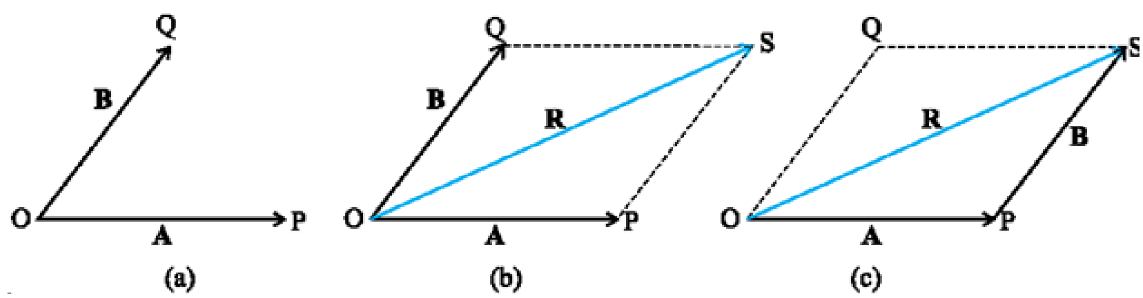
R_2 = (A – B).

The vector **R_1 = A + B** is also shown in the same figure for comparison

Parallelogram Law of Vector Addition

If two vectors A and B are represented both in magnitude and direction by the adjacent sides of a parallelogram drawn from the same point, their resultant is represented in magnitude and direction, by the diagonal of the parallelogram drawn from the same point.

We can also use the parallelogram method to find the sum of two vectors. Suppose we have two vectors A and B. To add these vectors, we bring their tails to a common origin O as shown below.



Then we draw a line from the head of A parallel to B and another line from the head of B parallel to A to complete a parallelogram OQSP. Now we join the point of the intersection of these two lines to the origin O. The resultant vector R is directed from the common origin O along the diagonal (OS) of the parallelogram Fig. (b)

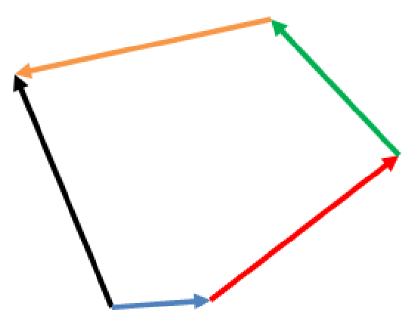
Also, the triangle law is used to obtain the resultant of A and B as shown in fig.(c) and we see that the two methods yield the same result.

Thus, the two methods are equivalent.

Polygon Law of Vector Addition

For more than two vectors, the triangle law can be extended to what is called the polygon law.

If a number of vectors are represented both in magnitude and direction by the sides of a polygon taken in the same order, their resultant is represented, in magnitude and direction, by the closing side of the polygon taken in the opposite order.

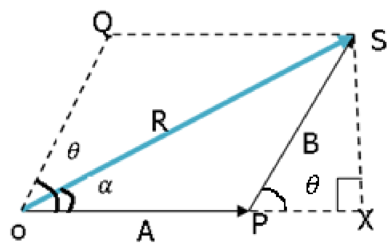


The black vector is the sum of all four vectors. Notice the placing of vectors for calculation may be used on a Google map to find the displacement from a start position to finish position.

Derivation of formula for Vector Addition

The graphical method is not very easy to use. It involves drawing diagrams where each vector quantity must be represented by a proportional vector and thus involves tedious drawing work, which in general, cannot be to the scale and therefore, lacks in accuracy. Another easier way to find the magnitude and direction of the resultant vector is the Analytical Method. It is an algebraic way to find the resultant of two vectors.

Let two vectors A and B, be represented in magnitude and direction by the sides OP and OQ of a parallelogram OPSQ, the angle between the vectors is θ as shown in fig. below, then from law of parallelogram of vector addition OS is the resultant. We will now see if we can find a mathematical relation between A, B, θ and R



Drop a perpendicular on extended line OP, let us mark the foot of the perpendicular from S on OP be X, to make a right angled triangle OXS

In ΔPXS,

$cos\ \theta$ = PX / PS or PX/B or PX = B $cos\theta$

$sin\ \theta$ = SX/PS or SX/B or SX = B $sin\theta$

In Δ OXS,

Using Pythagoras theorem as it is a right angled triangle

$$R^2 = OX^2 + SX^2$$

$R^2 = (A + PX)^2 + (SX)^2$

$$R^2 = (A + Bcos\theta)^2 + (Bsin\theta)^2$$

Thus, the magnitude of the resultant is:

$$|R| = \sqrt{(A^2 + (Bcos\theta)^2 + 2ABcos\theta) + (Bsin\theta)^2}$$

$$|\mathbf{R}| = \sqrt{(\mathbf{A^2} + (\mathbf{B})^2 + \mathbf{2ABcos\theta})}$$

Since, the resultant is a vector; it must also have a direction. The direction of R is given by the angle α between vector R and vector A.

Say α

So, $\tan\alpha = \frac{SX}{PX+A} = \frac{B\sin\theta}{A+B\cos\theta}$

The above is useful as it does not need any geometrical drawing with measurements, the formulae can be used to calculate resultant and the angle that the resultant makes with any one of the vectors.

If more than two vectors are to be added they can be taken two at a time and resultant of a pair can be taken with another vector to find that of the three vectors

Special cases

- If the two vectors are in the same line or the angle between them is 0

$$\boldsymbol{R} = \boldsymbol{A} + \boldsymbol{B}$$

- If the two vectors are perpendicular to each other θ = 90

  And $$R = \sqrt{A^2 + B^2}$$

- If the two vectors are at 180 °

$$\because \cos 180° = -1$$

  Then $\boldsymbol{R} = \boldsymbol{A} - \boldsymbol{B}$

EXAMPLE

Calculate the angle between a 2 N force and a 3 N force so that their resultant is 4 N.

SOLUTION

Using the formula derived above, we have

A = 2 N,

B = 3 N.

R = 4N

θ = ?

$$R = \sqrt{(\mathbf{A^2} + (\mathbf{B})^2 + \mathbf{2ABcos\theta})}$$

Putting the values and squaring both sides

16 = 4 + 9 + 2 x 2 x 3 $cos\ \theta$

⇒ $cos\ \theta$ = 3/12

⇒ $\boldsymbol{\theta}$ **= 75°**

EXAMPLE

The resultant of two equal forces acting at right angles to each other is 20 newton. Find the magnitude of either force.

SOLUTION

In this question, resultant R = 20 N, according to the question A = B and θ = 90°, cosθ =0

Using the formula for the resultant and putting all the data we have,

$$\mathrm{R} = \sqrt{(\mathbf{A^2} + (\mathbf{B})^2 + \mathbf{2ABcos\theta})}$$

$$20 \text{ x } 20 = 2\ A^2$$

$$\mathbf{A= 10\sqrt{2}\ newton}$$

EXAMPLE

A particle is acted upon by four forces simultaneously

(*i*) 30 N due east (*ii*) 20 N due North (*iii*) 50 N due West (*iv*) 40 N due South.

Find the resultant force on the particle.

SOLUTION

In all questions of this form, it will be wise to draw a simple diagram to show all the vectors in correct perspective as shown in diagram (a).

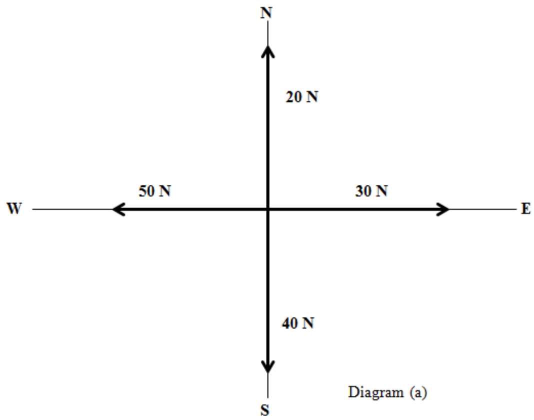


Diagram (a)

Then find the resultant of vectors which are collinear as in diagram (b).

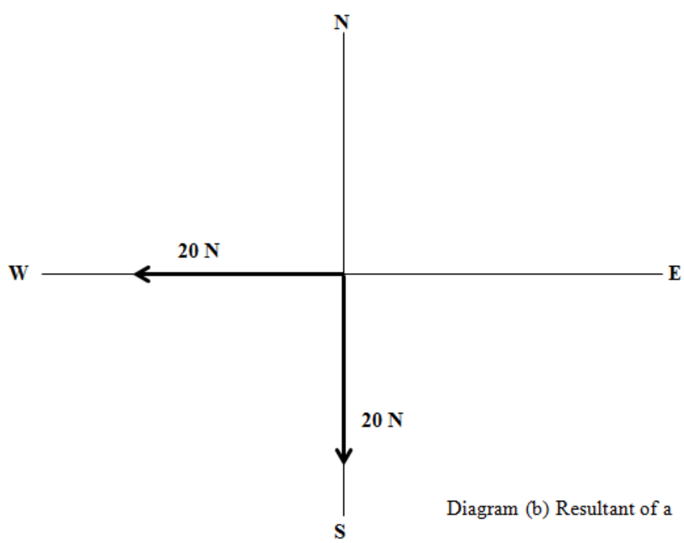


Diagram (b) Resultant of a

Now find the resultant using formula as in diagram (c).

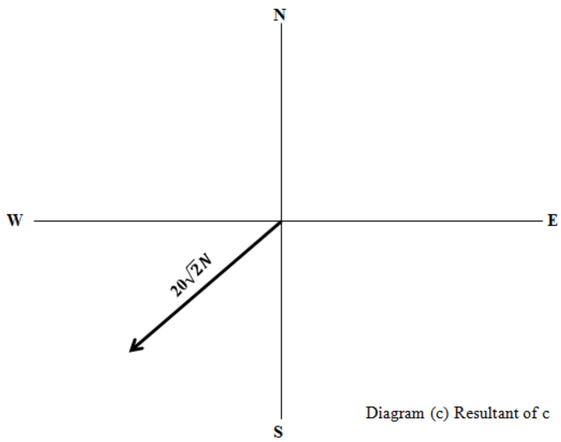


Diagram (c) Resultant of c

$$R = \sqrt{(\mathbf{20})^{\mathbf{2}} + (\mathbf{20})^{\mathbf{2}}}$$

$$= \mathbf{20\sqrt{2}}\ \boldsymbol{N}\ \textbf{along W to S}$$

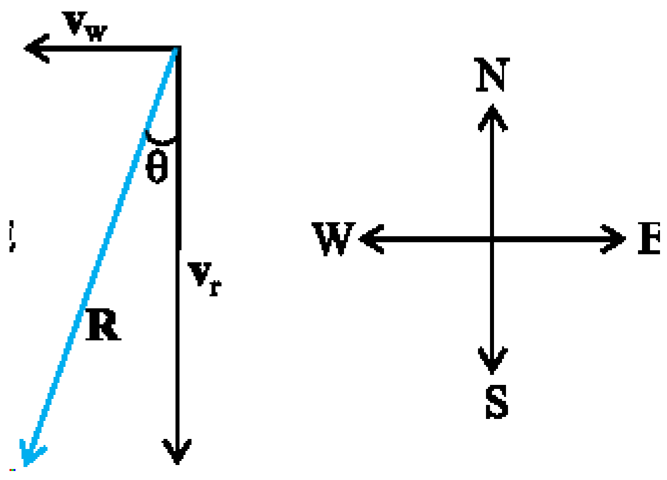
EXAMPLE

Rain is falling vertically with a speed of 35 m s^{-1}. Winds starts blowing after sometime with a speed of 12 m s^{-1} in east to west direction. In which direction should a boy waiting at a bus stop hold his umbrella?

SOLUTION

The velocity of the rain and the wind are represented by the vectors v_r and v_w

The vectors are drawn in the direction specified by the problem. Using the rule of vector addition, we see that the resultant of v_r and v_w is R as shown in the figure.



The magnitude of R

$$R = \sqrt{v_r^2 + v_w^2}$$

$$= \sqrt{35^2 + 12^2}$$

$$= 37 \text{ ms}^{-1}$$

The direction θ that R makes with the vertical is given by

$$tan\theta = \frac{v_w}{v_r} = \frac{12}{35} = 0.343$$

Or $\theta = tan^{-1}0.343 = 19°$

Therefore, the boy should hold his umbrella in the vertical plane at an angle of about 19° with the vertical towards the east.

EXAMPLE

A bus travelling at a speed of 20 m/s due south along a straight highway makes a left turn towards east keeping the speed same. It takes 25 second for the bus to completely change its direction of motion from south to east. Determine the magnitude of the average acceleration over the 25 second interval.

SOLUTION

The magnitude of the velocity here (note the word used is speed) remains the same but there is just a change in its directions. For speed, change in direction implies change in the velocity. This concept is highlighted in this question.

Although, the magnitude of velocity remains the same, the change in direction means velocity is changing.

Initial velocity = 20 m/s due South

Final velocity = 20 m/s due East

Change in velocity = $\boldsymbol{\Delta v = \sqrt{20^2 + 20^2}}$

Average rate of change of velocity

= Change in velocity/time

= $\mathbf{20\sqrt{2}}$ **m s^{-2}**

8. RESOLUTION OF A VECTOR

We can produce new vectors by vector addition and scalar multiplication. We call vector R the resultant of vectors A and B obtained by adding the two vectors A and B. Just as **R= A + B** can be construed as the sum of two other vectors, so can be each of the vectors A and B.

It is often convenient to break a vector to its components. Here the components of a vector are the vectors whose resultant equals the parent vector.

Basic or Unit Vector

The three Cartesian coordinate axes are represented by unit vectors $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$.

The utility of unit vectors will be clearer in the following topics

Components of Vectors

The process of finding a vector's components is known as "resolving", "decomposing", or "breaking down" a vector. Let's take the example of a vector A in a plane with a magnitude of *A* and making an angle θ with the positive *x*-axis.

The vector A can be written in terms of its two rectangular components as

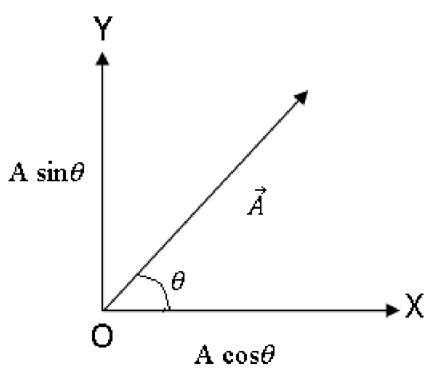
$$\mathbf{A} = \mathbf{A}\cos\theta\ \hat{\imath} + \mathbf{A}\sin\theta\ \hat{\jmath}$$

Where $\hat{\imath}$ *and* $\hat{\jmath}$ are the unit vectors along *x*-axis and *y*-axis respectively.

The components along the two axes can be written as

$\mathbf{A}_x = \mathbf{A}\cos\theta$

$\mathbf{A}_y = \mathbf{A}\sin\theta$ (as shown in fig. below)



We say so because, as we easily observe, the resultant of A_x and A_y equals A as per the triangle law of vector addition.

Therefore, $\mathbf{A} = \mathbf{A}_x\,\hat{\imath} + \mathbf{A}_y\,\hat{\jmath}$

The process can also be reversed, *i.e.*, if the components of a vector are known, we can get the magnitude of the vector and the angle it makes with *x*-axis as:

$$|\boldsymbol{A}| = \sqrt{\boldsymbol{A}_x^2 + \boldsymbol{A}_y^2}$$

The angle θ, which vector A makes with the x axis is

$$tan\theta = \frac{A_y}{A_x}$$

EXAMPLE

A vector of magnitude 20 makes an angle of 30° with the positive *x*-axis. What are its components? Express the vector in component form.

SOLUTION

Since the vector makes an angle of 30° with the *x*-axis, we can write

X – component, A_x= A cos θ

A_x= 20 cos 30° = 20 $\frac{\sqrt{3}}{2}$ = $10\sqrt{3}$

Y – component, A_y = A sin θ

A_y = 20 sin 30° = 20 x $\frac{1}{2}$ = 10

In component form the vector can be expressed as $\boldsymbol{A} = 10\sqrt{3}\hat{\imath} + 10\hat{\jmath}$

A given vector can be resolved into an infinite number of pairs of vectors in a plane.

It is to remove this ambiguity that we define the components of a vector along two well defined directions- the *x* and *y*-axis. Since the two axis are at right angles to each other we call this special pair of components as the 'rectangular components' of a given vector.

It is this 'the rectangular components' that is implied when we talk of the components of a given vectors.

Addition of Vectors using Components

Vector resolution is quite useful when you're asked to add two vectors that are neither parallel nor perpendicular. In such a case, one can resolve one vector into components that are parallel and perpendicular to the other vector.

EXAMPLE

Two persons are pulling a box in two directions. One person pulls due east with a force of 4 N. The second person pulls with a force of 8 N at an angle 30° west of north.

What is the total force acting on the box?

SOLUTION

To solve this problem, we need to resolve the force of the second person into its northward and westward components.

Because the force is directed 30° west of north, its northward component is

F_y= 8 cos 30° = 8.0 × 0.86 = 6.8 N

and its westward component is

F_x = 8 sin 30° = 8.0 × 0.5 = 4 N

Since the eastward component is also 4.0N, the eastward and westward components cancel one another out. **The resultant force is directed due north, and equals of approximately 6.8 N.**

TRY YOURSELF:

- **Can the magnitude of the rectangular component of a vector be greater than the magnitude of the vector itself?**
- **What is the maximum number of components into which a vector can be resolved?**

EXAMPLE

A vector is resolved into two 'components' each of which has the same magnitude as the vector itself. What is the angle between the two directions along which the vector?

SOLUTION $A = A_x = A_y$ ……….. (1)

$$\boldsymbol{A = \sqrt{A_x^2 + A_y^2 + 2A_xA_y cos\theta}}$$

Using (1) and squaring both sides, we get

$$\boldsymbol{A^2} = \mathbf{2}\boldsymbol{A^2}(\mathbf{1} + \mathbf{cos}\,\boldsymbol{\theta})$$

$$(\mathbf{1} + \boldsymbol{cos\theta}) = \frac{\mathbf{1}}{\mathbf{2}}$$

$\Rightarrow cos\ \theta = -1/2$

$\Rightarrow \theta = 120^0$

9. MULTIPLICATION OF A VECTOR BY A VECTOR- SCALAR PRODUCT AND CROSS PRODUCT

Recall that multiplication is, in scalar algebra, like repeated addition. Multiplying 4 by 3 means adding four three times: 4 × 3 = 4 + 4 + 4=12. The multiplication of a vector by a scalar works in the same way. Multiplying the vector *A* by a positive scalar *c* is equivalent to adding together *c* copies of the vector *A*.

Thus, 3*A* = *A* + *A* + *A*.

Multiplying a vector by a positive scalar will result in a vector with the same direction as the original, but of a magnitude that is the scalar times the magnitude of the vector itself.

VECTOR MULTIPLICATION

Mathematics is the language of science. It is used to interpret and predict the nature around us. We use mathematical operations in a particular way to explain and interpret several phenomenon. The choice of selection of mathematical tool is in our hands

So, apart from vector addition and subtraction, there are two other forms of combining a pair of vectors: scalar product and vector product. The first one results in a scalar, and the second one results in a vector. We define these two types of multiplications for vectors keeping in view the nature of physical quantities that result from the multiplication of two (physical) vector quantities.

SCALAR OR DOT PRODUCT

A number of physical quantities are the result of multiplication of two vectors but are themselves scalars, *e.g.*, work,

Such products of vector quantities are specified through their scalar product, or the dot product.

Work is a scalar quantity, but it is measured by the magnitude of force and displacement, (both vector quantities), and the degree to which the force and displacement are inclined to one another.

The dot product of any two vectors, *A* and *B*, is expressed by the equation

A.B = AB cos θ

Geometrical meaning of scalar product A.B = AB cos θ = A (B cos θ) = (A cos θ) B

The dot product of *A* and *B* is the value you would get by multiplying the magnitude of *A* by the magnitude of the component of *B* in the direction of *A*.

We get the same result if we multiply the magnitude of *B* by the magnitude of the component of *A*, which lies in the direction of *B*. Here θ, is the angle between A and B

PROPERTIES OF SCALAR PRODUCT

i. **A.B = B.A**

ii. **A.(B+C)=(A+B).C**

iii. **A.B is equal to the sum of the products of their corresponding rectangular components**

$A.B = A_xB_x + A_yB_y + A_zB_z$

The cosine of the angle between A and B is given by

$$\cos\theta = \frac{A.B}{|A||B|} = \frac{A_xB_x+A_yB_y+A_zB_z}{\sqrt{A_x^2+A_y^2+A_z^2}\sqrt{B_x^2+B_y^2+B_z^2}}$$

iv. $\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$; $i \times j = k, j \times k = i, k \times i = 0$

VECTOR OR CROSS PRODUCT

The vector product, also called the cross product, “multiplies” two vectors to produce a third vector, which is perpendicular to the plane defined by both of the original vectors. We use this definition of the cross product of two vectors, for defining physical quantities like torque of a force' or the 'magnetic field of a current carrying conductor.

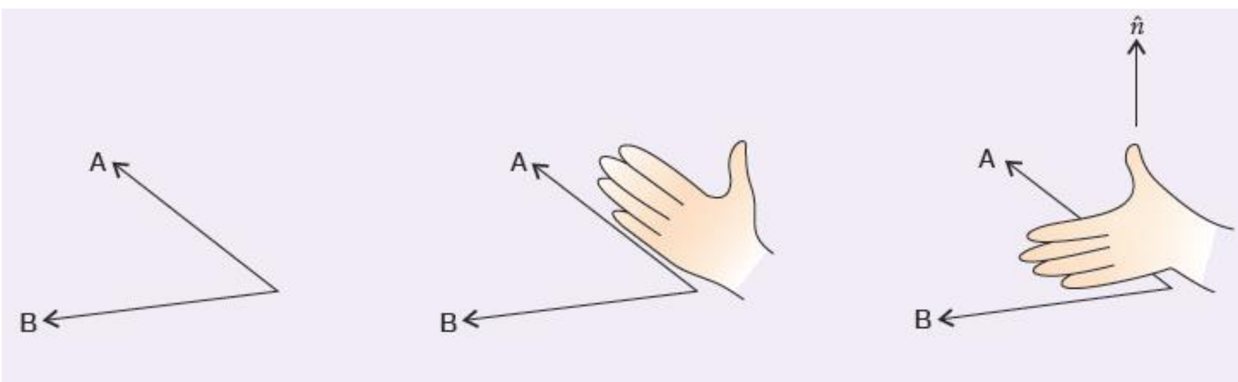
The cross product demands us to think in three dimensions. The cross product of two vectors, *A* and *B*, is defined by the equation:

$\mathbf{A} \times \mathbf{B} = (\mathbf{AB} \sin \theta)\, \hat{\boldsymbol{n}}$

Where $\hat{n}$ is a unit vector perpendicular to both *A* and *B*. This required $\hat{n}$ to be along the normal to the plane defined by A and B.

The normal to a plane, can be directed 'upwards' or 'downwards'. To remove this ambiguity, we define the direction of n-through the so called right–hand rule.

The right-hand rule is as follows: In order to find the direction of a cross product A× B, align your right hand along the first vector *A*, then curl your fingers via the small angle toward the vector *B*. The direction in which your thumb is pointing is the direction of *n*, *i.e.*, the direction of A×B.



An alternative form of the 'right hand rule' is as follows:

Imagine a screw to be held in the right hand along the normal to the plane defined by A and B. Let the screw be rotated along the sense: 'from A to B'. The direction of the cross product A × B is the direction in which the screw would advance.

Properties of Vector Product:

The Vector product, of two vectors has the following properties:

(i) **$\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A}$**

The vector product of two vectors thus does not obey the commutative law. The order of multiplication becomes important here.

(ii) **(A + B) × C = A × C + B × C**

The vector product of two vectors thus obeys the 'distributive law' of multiplication.

(iii) **A × A = 0**

This is an interesting property. The cross product of a vector with itself thus yields the 'zero vector' as its result.

(iv) **If A ≠ 0, B ≠ 0, but A × B = 0**

This implies that if the cross product of two non-zero, vectors can be zero.

(v) $\boldsymbol{i} \times \boldsymbol{i} = \boldsymbol{j} \times \boldsymbol{j} = \boldsymbol{k} \times \boldsymbol{k} = 0$

(vi) $\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k},\ \boldsymbol{j} \times \boldsymbol{k} = \boldsymbol{i},\ \boldsymbol{k} \times \boldsymbol{i} = \boldsymbol{j}$

(vii) $\boldsymbol{j} \times \boldsymbol{i} = -\boldsymbol{k},\ \boldsymbol{k} \times \boldsymbol{j} = -\boldsymbol{i},\ \boldsymbol{i} \times \boldsymbol{k} = -\boldsymbol{j}$

Vector Product in terms of Rectangular Components

(Determinant Method)

Let $\mathbf{A} = A_x\boldsymbol{i} + A_y\boldsymbol{j} + A_z\boldsymbol{k}$**, and** $\mathbf{B} = B_x\boldsymbol{i} + B_y\boldsymbol{j} + B_z\boldsymbol{k}$

$\mathbf{A} \times \mathbf{B} = 0 + A_xB_y(\boldsymbol{k}) + A_xB_z(-\boldsymbol{j}) + A_yB_x(-\boldsymbol{k}) + 0 + A_yB_z(\boldsymbol{i}) + A_zB_x(\boldsymbol{j}) + A_zB_y(-\boldsymbol{i}) + 0$

$= \boldsymbol{i}(A_yB_z - A_zB_y) - \boldsymbol{j}(A_xB_z - A_zB_x) + \boldsymbol{k}(A_xB_y - A_yB_x)$

This can be put in the 'easy to remember' determinant form: $\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

EXAMPLE

Find the:

a) Dot product

b) Cross product of

A = i – j +k and B = 2 i +j + k

SOLUTION

a) Dot product A.B = $\frac{2-1+1}{\sqrt{3}\sqrt{6}} = \frac{2}{\sqrt{18}}$

b) Cross product using the determinant method

$$\begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

A × B = -2i +j +3k

10. SUMMARY

In this module we have learnt:

- Vectors have both magnitude and direction
- Vectors can be used to represent physical quantities
- Vectors can be added and subtracted by geometrical and analytical methods
- Vectors can be resolved into rectangular components making vector algebra simpler
- Vectors can be multiplied in two ways
- If the product of vectors is a scalar we call it scalar product or dot product

  A.B = AB cos θ

- If the product of vectors is a vector we call it vector product or cross product

  A × B = AB sin θ